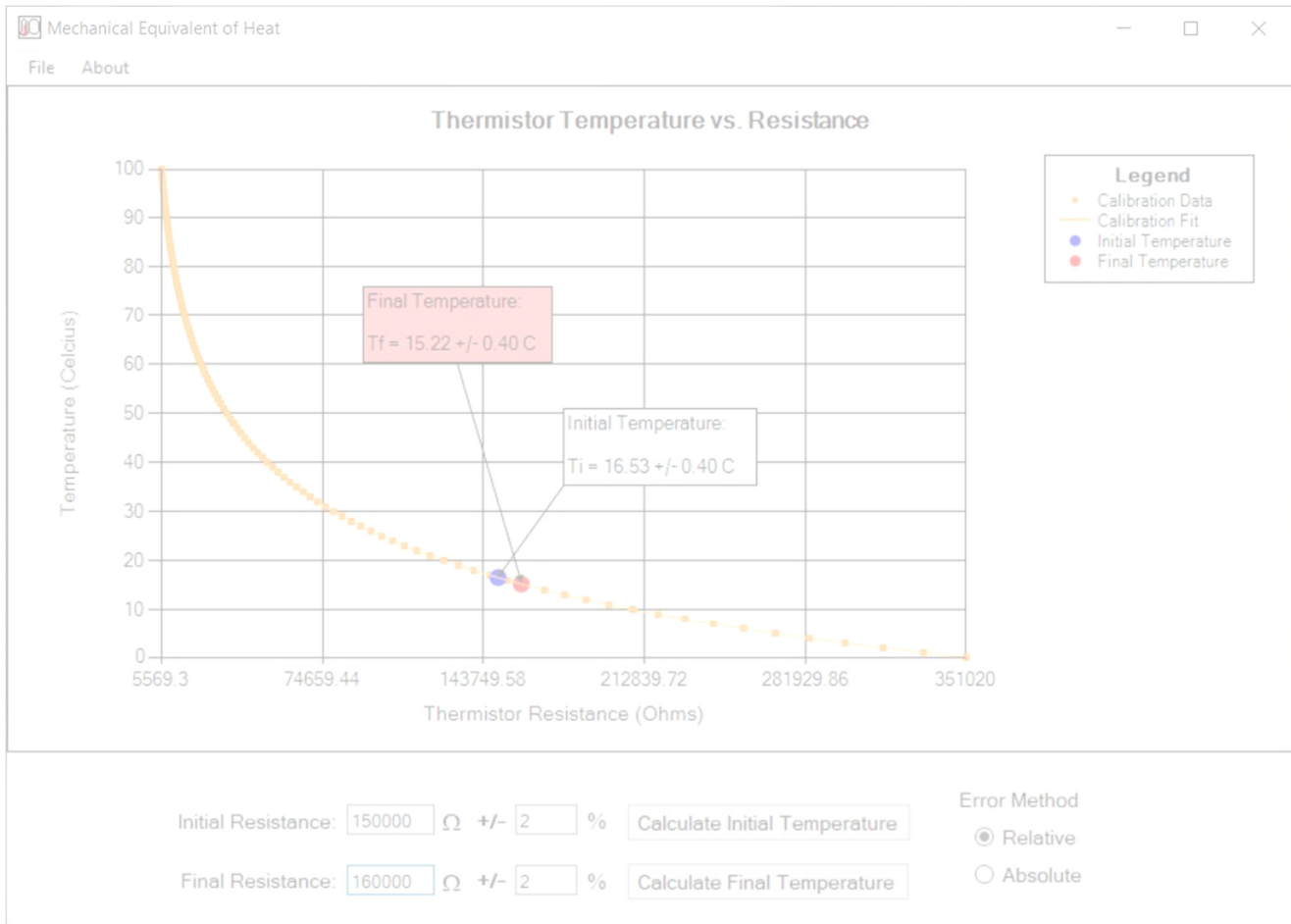# Mechanical Equivalent of Heat
ver. 2.0

Users Guide

Introduction

Mechanical Equivalent of Heat (MEH) calculates the temperature of a thermistor based on a user-supplied resistance. The user provides a plain ASCII calibration file containing temperature and resistance data for the thermistor to be used. MEH plots the provided calibration data and performs a least-square fit to determine the Steinhart-Hart coefficients[1] (with uncertainties) for the thermistor being used. Once calibrated, MEH will calculate the temperature based on a user-supplied resistance.

MEH was developed for use with PASCO Scientifics' "Mechanical Equivalent of Heat" apparatus, but it can be used with any thermistor provided the user supplies the data file for the calibration. By default, MEH uses the temperature vs. resistance data furnished by PASCO scientific[2] for use with the thermistor embedded in the aluminum cylinder of PASCO's "Mechanical Equivalent of Heat" apparatus.
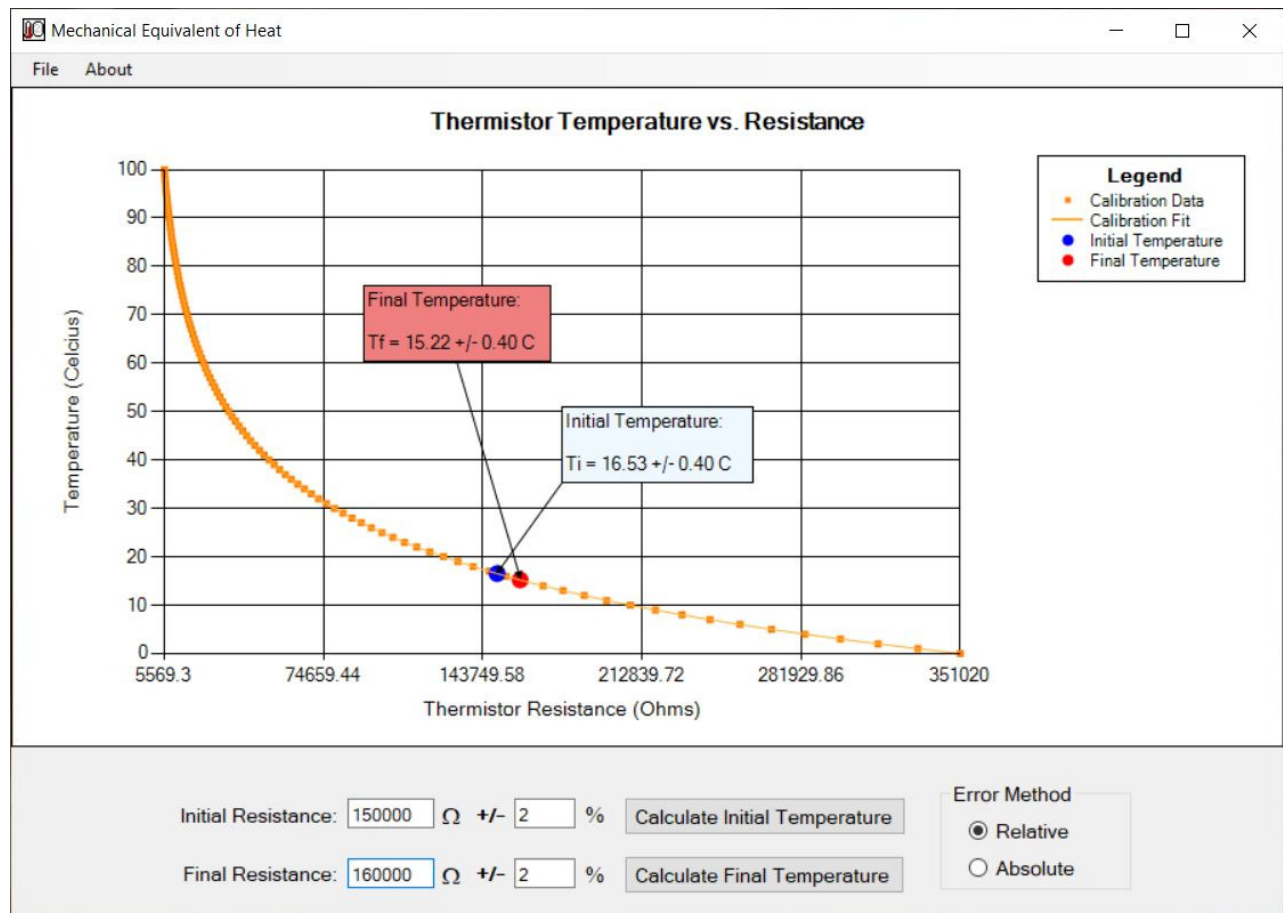

Figure 1: Screenshot of MEH.

How to Use MEH

To determine a temperature using MEH:

When MEH runs, it looks for a file named "MEH_Input.txt" containing the temperature vs. resistance data for the thermistor being used. The file is opened, and a least-square fit is performed on that data in order to determine the Steinhart-Hart coefficients. If the file does not exist, it is created using the thermistor data provided by PASCO[2] Scientific. To use MEH, enter the measured resistance into either resistance box and click the corresponding Calculate Temperature button. (See example in Figure 2).
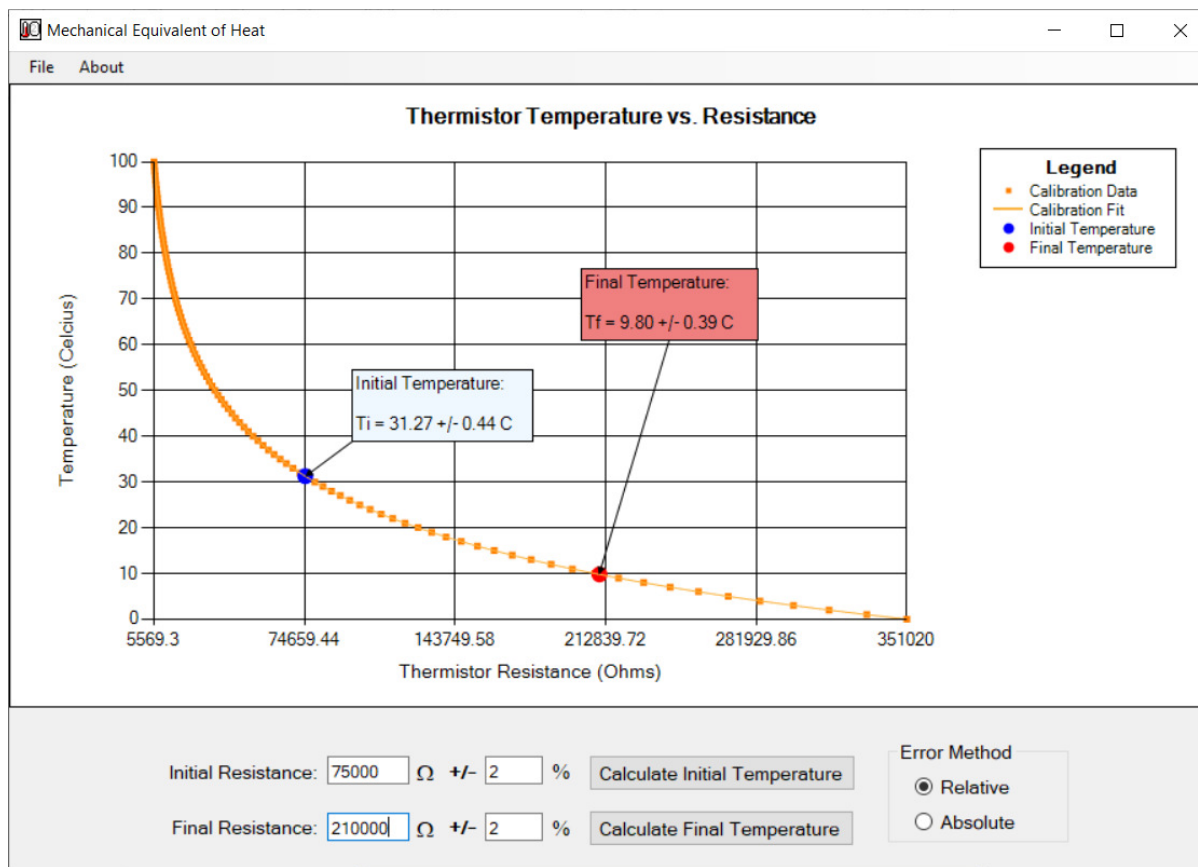

Figure 2: Example of using MEH. A resistance of $75,000(\pm 2\%)\Omega$ gives an initial temperature of (31.27 +/- 0.44) ºC, and a resistance of $210,000(\pm 2\%)\Omega$ gives an final temperature of (9.80 +/- 0.39) ºC.

Using Your Own Thermistor Input File

The format of the input text file is a plain ASCII text file with one TAB delimited temperature, resistance data point per line. The resistance must be in ohms and the temperature in ^oC. See Figure 3 for an example. Be sure to name the calibration file "MEH_Input.txt" and place it in the same directory as the MEH application.

```
0    351020
1    332640
2    315320
3    298990
4    283600
5    269080
6    255380
7    242460
8    230260
9    218730
10   207850
11   197560
12   187840
13   178650
```

Figure 3: Format of input data file showing resistance and temperature separated by a TAB. For example, $283,600 \ \Omega$ corresponds to 4^oC.

Viewing Steinhart-Hart Coefficients

The Steinhart-Hart coefficients can be viewed at any time by choosing **Steinhart Coefficients** from the MEH file menu.
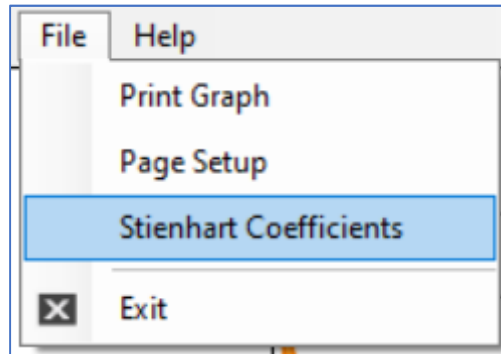

Figure 4: Viewing the Steinhart-Hart coefficients.

The fit results for the included "MEH_Input.txt" file are shown below in Figure 5.

Steinhart-Hart Coefficients

8.267E-004 +/- 1.975E-007
2.089E-004 +/- 2.812E-008
8.033E-008 +/- 8.194E-011

Figure 5: Sample coefficients.

The Gory Details

The relationship between resistance and temperature for a thermistor is described by the Steinhart-Hart[1] equation:

$$\frac{1}{T_K} = A + B\ln(R) + C\left(\ln(R)\right)^3$$

where T_K is the absolute temperature in Kelvin and R is the resistance in Ohms. The coefficients A, B and C are the Steinhart-Hart coefficients and vary depending on the thermistor. Defining $x = \ln(R)$ and $y = \frac{1}{T_K}$, the equation above can be rewritten in the form:

$$y = A + Bx + Cx^3$$

The coefficients A, B and C can now be determined using a standard least-square fit approach[3]. For a set of calibration data (x_i, y_i), we want to find the values of A, B and C that minimize χ^2:

$$\chi^2 = \sum \left(\frac{y_i - A - Bx_i - Cx_i^3}{\delta y_i} \right)^2$$

where δy_i is the uncertainty in y_i. Assuming δy_i to be the same for all y_i, we then minimize χ^2 with respect to each coefficient A, B and C to obtain a set of three equations and three unknowns[i]. After a bit of algebra, one can write the set of equations as the matrix equation:

$$\begin{pmatrix} N & \sum x_i & \sum x_i^3 \\ \sum x_i & \sum x_i^2 & \sum x_i^4 \\ \sum x_i^3 & \sum x_i^4 & \sum x_i^6 \end{pmatrix} \begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} \sum y_i \\ \sum x_i y_i \\ \sum x_i^3 y_i \end{pmatrix}$$

Using Maple© to solve this set of equations (via Cramer's Rule) gives:

$$A = \frac{\sum x_i \sum x_i y_i \sum x_i^6 - \sum x_i \sum x_i^3 y_i \sum x_i^4 - \sum x_i^3 \sum x_i y_i \sum x_i^4 + \sum x_i^3 \sum x_i^3 y_i \sum x_i^2 + \sum y_i \left(\sum x_i^4\right)^2 - \sum y_i \sum x_i^2 \sum x_i^6}{-2\sum x_i^3 \sum x_i \sum x_i^4 + N\left(\sum x_i^4\right)^2 + \left(\sum x_i^3\right)^2 \sum x_i^2 + \left(\sum x_i\right)^2 \sum x_i^6 - N\sum x_i^6 \sum x_i^2}$$

$$B = \frac{-\sum x_i \sum x_i^3 y_i \sum x_i^3 + \sum x_i \sum y_i \sum x_i^6 - \sum x_i^3 \sum y_i \sum x_i^4 - N\sum x_i y_i \sum x_i^6 + \sum x_i y_i \left(\sum x_i^3\right)^2 + N\sum x_i^3 y_i \sum x_i^4}{-2\sum x_i^3 \sum x_i \sum x_i^4 + N\left(\sum x_i^4\right)^2 + \left(\sum x_i^3\right)^2 \sum x_i^2 + \left(\sum x_i\right)^2 \sum x_i^6 - N\sum x_i^6 \sum x_i^2}$$

and

---

[i] Calculating $\frac{\partial \chi^2}{\partial A} = 0$, $\frac{\partial \chi^2}{\partial B} = 0$ and $\frac{\partial \chi^2}{\partial C} = 0$ give three equations and three unknowns that can be solved to obtain A, B and C.

$$C = \frac{-\left(\sum x_i \sum x_i y_i \sum x_i^3 + \sum x_i \sum y_i \sum x_i^4 - N\sum x_i^4 \sum x_i y_i - \sum x_i^3 \sum y_i \sum x_i^2 - \sum x_i^3 y_i \left(\sum x_i\right)^2 + N\sum x_i^3 y_i \sum x_i^2\right)}{-2\sum x_i^3 \sum x_i \sum x_i^4 + N\left(\sum x_i^4\right)^2 + \left(\sum x_i^3\right)^2 \sum x_i^2 + \left(\sum x_i\right)^2 \sum x_i^6 - N\sum x_i^6 \sum x_i^2}$$

To determine uncertainties in the coefficients, uncertainties in x_i are considered negligible, and the uncertainties in y_i are assumed to be the same for all y_i. The uncertainty in y_i is then estimated from the sample variance of y. That is,

$$\delta y = \frac{1}{N-3}\sqrt{\left(y_i - A - Bx_i - Cx_i^3\right)^2}$$

for N calibration data points. Finally, the uncertainty in the coefficients can be calculated from the estimated uncertainties y_i to obtain:

$$\delta A = \sqrt{\sum\left(A - A_{\delta y_i}\right)^2} \text{ where } A_{\delta y_i} \equiv A(y_i + \delta y),$$

$$\delta B = \sqrt{\sum\left(B - B_{\delta y_i}\right)^2} \text{ where } B_{\delta y_i} \equiv B(y_i + \delta y), \text{ and}$$

$$\delta C = \sqrt{\sum\left(C - C_{\delta y_i}\right)^2} \text{ where } C_{\delta y_i} \equiv C(y_i + \delta y).$$

Using this technique to fit the data supplied by PASCO returns the coefficients $A = (8.267 \pm 0.002) \times 10^{-4}\,\text{K}^{-1}$, $B = (2.089 \pm 0.0003) \times 10^{-4}\,\text{K}^{-1}$, and $C = (8.033 \pm 0.008) \times 10^{-8}\,\text{K}^{-1}$. With these values for the Steinhart-Hart coefficients, the temperature of the aluminum cylinder is calculated from the user-supplied value of the resistance.

The total uncertainty in the temperature (δT) is calculated by propagating the contributing uncertainties from A, B, C, and R using Squires' method again. For example, the contribution to δT due to the uncertainty in the A coefficient is given by $(T - T_{\delta A})$ where $T_{\delta A} \equiv T(A + \delta A)$. That is:

$$T_{\delta A} \equiv \frac{1}{(A + \delta A) + B\ln(R) + C\left(\ln(R)\right)^3}$$

The contribution to δT due to the error in B is given by $(T - T_{\delta B})$ where:

$$T_{\delta B} \equiv \frac{1}{A + (B + \delta B)\ln(R) + C\left(\ln(R)\right)^3}$$

Defining similar expressions for $T_{\delta C}$ and $T_{\delta R}$, δT is finally determined:

$$\delta T = \sqrt{\left(T - T_{\delta A}\right)^2 + \left(T - T_{\delta B}\right)^2 + \left(T - T_{\delta C}\right)^2 + \left(T - T_{\delta R}\right)^2}$$

References

1. Steinhart, J. S. and S. R. Hart, "Calibration curves for thermistors", Deep Sea Res., **15**, 497-503 (1968).

2. "Instruction Manual and Experiment Guide for the PASCO scientific Model TD-8551A", PASCO Scientific, 9 (May 1994).

3. G.L. Squires, Practical Physics, 4th ed. (Cambridge University Press, 2001), p. 46 – 48.